Mortality and Income Inequality

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Link between aggregate life expectancy and aggregate income?

- Guess: more income extends our lives $\rightarrow$ income $\uparrow$ in life expectancy (LE)
- How to explain that US GDP/capita is higher than Great Britain (GBR) GDP/capita, but the reverse is true when it comes to life expectancy (LE)?
- Also if the population effects productivity, then LE $\uparrow$ feeds back into income...
- This project: Inequality in productivity generates inequality in mortality
  - Can decouple aggregate LE and GDP/capita, while still maintaining that LE is increasing in income at an individual level
  - Recognize that distribution of income and convexity of mapping from income to mortality are key
  - US vs. GBR Data consistent with (at least one interpretation of) model

Agents are born, eat to stave off death, and die

- Agents are born at rate b, and draw permanent $z \sim F$
- Die at rate $\delta c(z)^{-\gamma}$, where $c(z)$ is consumption
  - γ: elasticity of z-population to consumption
- Use unit of labor to produce $y(z) = A(N)z$ goods, where N is population size, $A(N) = \overline{A}N^{\beta}$
  - β: elasticity of aggregate productivity to population
  - $\beta < 0$: “crowding out productivity" (Malthusian)
  - $\beta > 0$: “more people means more productive", e.g. via more specialization
- Hand-to-mouth, so $c(z) = y(z)$

Population size increasing in γ-th moment of productivity

- Population $N(z)$ for z type follows $\dot{N}(z) = b - \delta(A(N)z)^{-\gamma}N(z)$
- Steady-state $N^*(z) = \frac{b}{\delta}(A(N)z)^{\gamma}$
- Aggregate population is then

$$N = \int f(z)N(z)\mathrm{d}z$$
$$= \left(\overline{A}^{\gamma}\frac{b}{\delta}\int f(z)z^{\gamma}\mathrm{d}z\right)^{\frac{1}{1-\beta\gamma}}$$

- Elasticity of N to $\mathbb{E}[z^{\gamma}]^{\frac{1}{\gamma}}$ is $\frac{\gamma}{1-\beta\gamma}$: $\beta > 0$ amplifies pop response to shifts in z-dist

Life expectancy increasing in γ-th moment of productivity

- Life expectancy at birth is

$$
\begin{aligned}
LE &= \int f(z) \frac{1}{\delta c(z)^{-\gamma}} \mathrm{d}z \\
&= \int f(z) \delta^{-1} (AN^{\beta} z)^{\gamma} \mathrm{d}z \\
&= \delta^{-1} A^{\gamma} N^{\beta\gamma} \int f(z) z^{\gamma} \mathrm{d}z \\
&= \delta^{-1} A^{\gamma} \left(A^{\gamma} \frac{b}{\delta} \int f(z) z^{\gamma} \mathrm{d}z \right)^{\frac{\beta\gamma}{1-\beta\gamma}} \int f(z) z^{\gamma} \mathrm{d}z \\
&= \left(\overline{A}^{\gamma} \frac{b^{\beta\gamma}}{\delta} \int f(z) z^{\gamma} \mathrm{d}z \right)^{\frac{1}{1-\beta\gamma}}
\end{aligned}
$$

- Note: birth rate b affects LE, because more births may stimulate or depress productivity

Income depends on γ-th and $(1+\gamma)$-th moments of productivity

- Average income of population is weighted by $f(z)N(z)$, not $f(z)$, so selects for more productive, longer living types

$$\mathbb{E}[y(z)] = \int \frac{f(z)N(z)}{N} y(z)\mathrm{d}z$$

$$= \left(\frac{b}{\delta}\right)^{\frac{\beta}{1-\beta\gamma}} \overline{A}^{1+\frac{\beta\gamma}{1-\beta\gamma}} \underbrace{\left(\int f(z)z^{\gamma}\mathrm{d}z\right)^{\frac{\beta}{1-\beta\gamma}}}_{\text{Pop size affects productivity}} \frac{\overbrace{\int f(z)z^{1+\gamma}\mathrm{d}z}^{\text{Selection towards more productive survivors}}}{\underbrace{\int f(z)z^{\gamma}\mathrm{d}z}_{\text{Less people to share agg income}}}$$

Changes in z dist may lower LE and raise income/capita...

- Consider shift $F \to \tilde{F}$ such that $\mathbb{E}_F[z^\gamma] > \mathbb{E}_{\tilde{F}}[z^\gamma]$ but $\mathbb{E}_F[z^{1+\gamma}] < \mathbb{E}_{\tilde{F}}[z^{1+\gamma}]$ (e.g. mean-preserving spread)
- If $\beta < 0$ (higher pop helps agg prod)
  - LE $\propto \mathbb{E}[z^\gamma]^{\frac{1}{1-\beta\gamma}} \downarrow$
  - Income/capita $\propto \mathbb{E}[z^\gamma]^{\frac{\beta}{1-\beta\gamma}} \frac{\mathbb{E}[z^{1+\gamma}]}{\mathbb{E}[z^\gamma]} \uparrow$

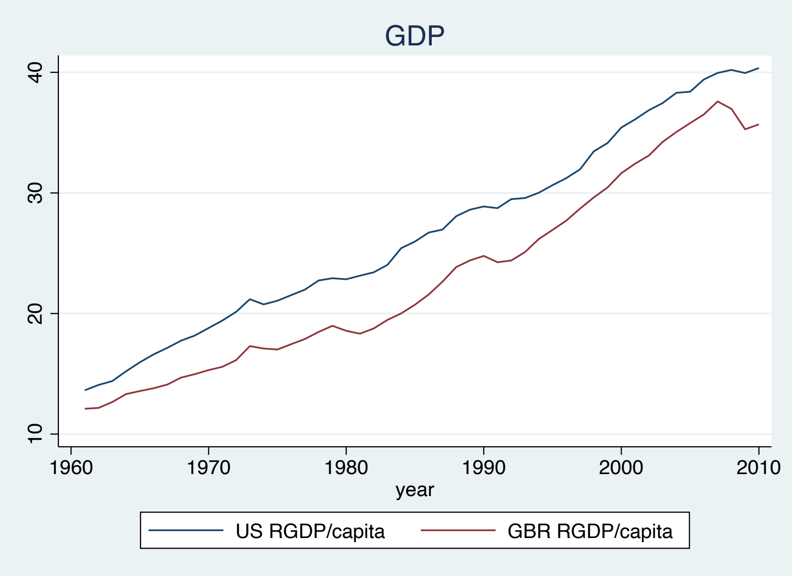
...or may lower LE and have ambiguous effect on income/capita

- Consider shift $F \to \tilde{F}$ such that $\mathbb{E}_F[z^\gamma] > \mathbb{E}_{\tilde{F}}[z^\gamma]$ but $\mathbb{E}_F[z^{1+\gamma}] < \mathbb{E}_{\tilde{F}}[z^{1+\gamma}]$ (e.g. mean-preserving spread)
- If $\beta > 0$ (higher pop helps agg prod)
  - LE $\propto \mathbb{E}[z^\gamma]^{\frac{1}{1-\beta\gamma}} \downarrow$
  - Income/capita $\propto \mathbb{E}[z^\gamma]^{\frac{\beta}{1-\beta\gamma}} \frac{\mathbb{E}[z^{1+\gamma}]}{\mathbb{E}[z^\gamma]}$ ambiguous: depends on degree of amplification via β versus concavity γ
- Rise in inequality can lower income/capita by lowering the population enough to override the selection towards more productive workers
- As $\gamma \to 1$, $\mathbb{E}[z^\gamma]$ becomes less sensitive to mean preserving spreads of γ

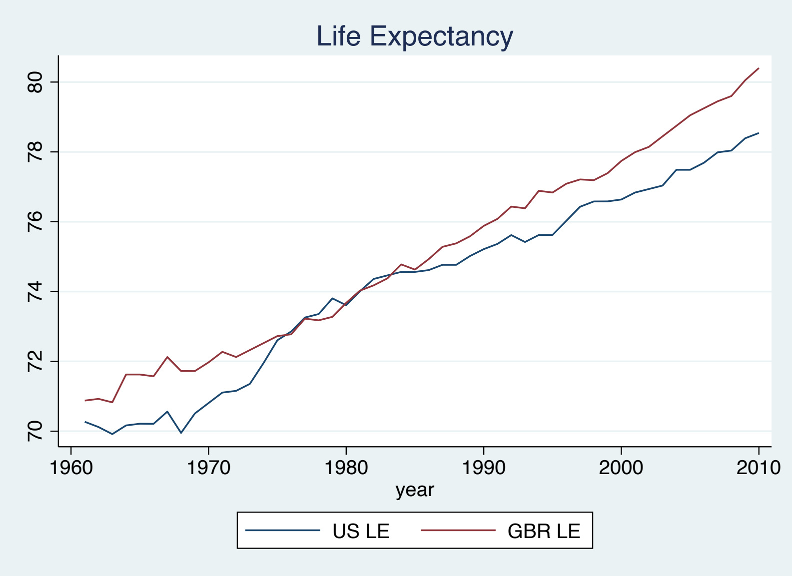
Predict US LE should respond more to income inequality than GBR

- Some evidence lower US LE driven largely by young and poor dying at a higher rate than in GBR (Deaton and Paxson (2004), Pritchard (2021))
- Also some evidence that differences in LE between income groups has increased over time (Chetty et al (2016))
- This sounds like the mapping from income to death rates is more convex in the US than in GBR, i.e. lower γ, or that it has perhaps become more convex over time
- Then as US income inequality shifts, the lower γ will mean LE is more sensitive than in GBR (after controlling for mean income effects)

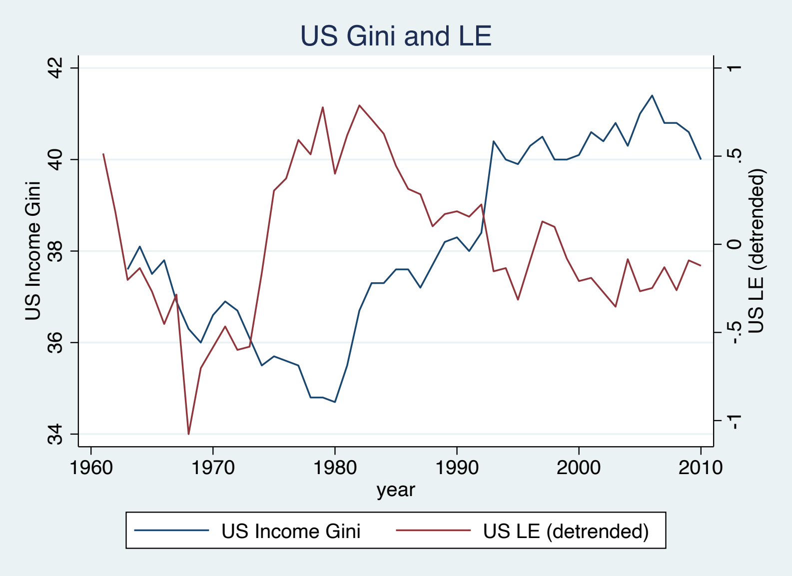
US has higher RGDP/capita than GBR



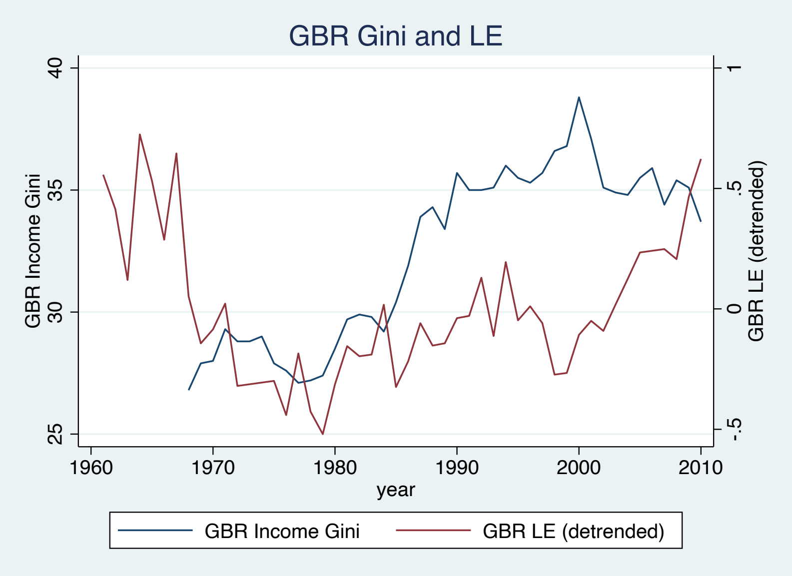
US has lower life expectancy than GBR



US income inequality seems to (inversely) track LE



GBR income inequality seems to track LE much less



US LE is more (negatively) responsive to income inequality

	(1) US LE (detrended)	(2) GBR LE (detrended)	(3) US LE	(4) GBR LE	(5) US LE	(6) GBR LE
US Income Gini	-0.06*		-0.22***		-0.21***	
	(0.02)		(0.00)		(0.00)	
GBR Income Gini		0.03***		-0.02		0.09*
		(0.00)		(0.39)		(0.04)
year			0.21***	0.21***		
			(0.00)	(0.00)		
US RGDP/capita					0.37***	
					(0.00)	
GBR RGDP/capita						0.31***
						(0.00)

p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Conclusion: theory has bite!

- Theory linking mortality with income
  - Mortality distribution determines population, which affects productivity, thus income distribution
  - Income distribution affects mortality distribution
- Dispersion in productivity distribution lower LE, but may raise or lower income/capita
- Convexity of mapping from income to mortality key for determining how z-dist shifts will affect LE
- Differences in concavity between US and GBR qualitatively consistent with time trends: US LE more tightly (and negatively) tracks inequality